New (and New New) Trade Theory: Armington versus Krugman versus Melitz

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1 The Melitz (2003) theory revisited:

For good i in region r take as given demand for the composite of domestic and foreign varieties, which is a function of the general-equilibrium price vector:

$$Q_{ir} = Q_{ir}(\boldsymbol{P}) \qquad \perp P_{ir}. \tag{1}$$

Also take as given the "output" level of good i in region s, which is also a function of the general-equilibrium price vector:

$$Y_{is} = Y_{is}(\boldsymbol{P}) \qquad \perp Y_{is}. \tag{2}$$

In the monopolistic competitions theories we will interpret Y_{is} as the "supply" of a single composite input.

Table 1: Notation and Variable Definitions

Variable	**Armington**	**Krugman**	**Melitz**
Composite-commodity demand	Q_{jr}	Q_{kr}	Q_{hr}
Price index on composite commodity	P_{jr}	P_{kr}	P_{hr}
Number of entered firms			M_{hr}
Number of active firms		N_{kr}	N_{hrs}
Firm-level output		q_{krs}	$\tilde{q}_{hrs}$
Firm-level price		p_{krs}	$\tilde{p}_{hrs}$
Firm-level productivity			$\tilde{\varphi}_{hrs}$
Composite-input unit cost	c_{jr}	c_{kr}	c_{hr}
Composite-input supply	Y_{jr}	Y_{kr}	Y_{hr}

1.1 Melitz (Model A3)

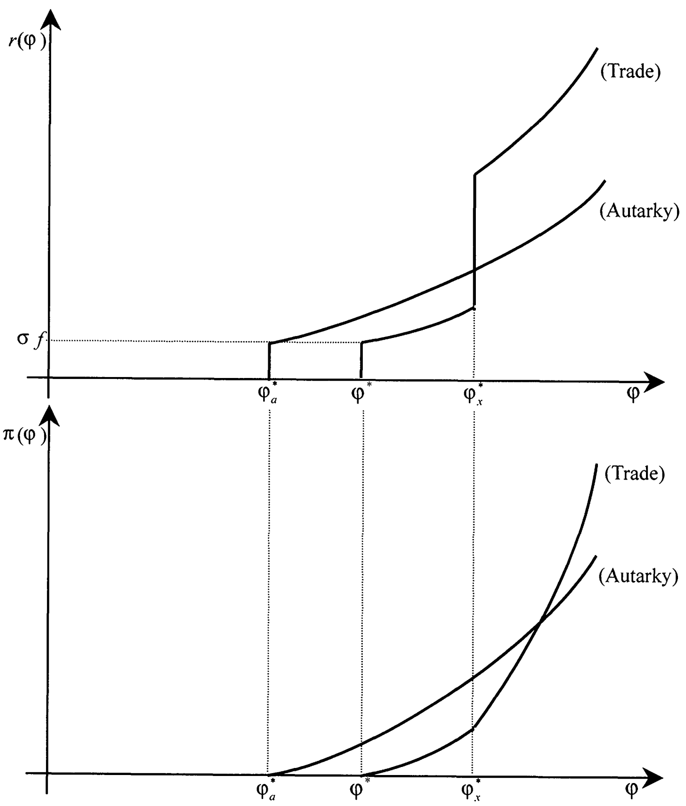

FIGURE 2.—The reallocation of market shares and profits.

Firm specific productivity is given by φ where cost equals $f + q/\varphi$. The productivity level is drawn from a Pareto distribution with shape parameter a and lower support b.
PDF:

$$g(\varphi) = \frac{a}{\varphi}\left(\frac{b}{\varphi}\right)^a$$

This allows Melitz to define the representative firm (drawing $\tilde{\varphi}$) and relate this firm to the marginal firm.

1.2 Melitz (Model A3)

$$P_{hs} = \left[\sum_r N_{hrs}\tilde{p}_{hrs}^{1-\sigma_h}\right]^{1/(1-\sigma_h)} \qquad \perp Q_{hs} \tag{3}$$

$$\tilde{q}_{hrs} = Q_{hs}\left(\frac{P_s}{\tilde{p}_{hrs}}\right)^{\sigma_h} \qquad \perp \tilde{p}_{hrs} \tag{4}$$

$$\tilde{p}_{hrs} = \frac{c_{hr}\tau_{hrs}}{\tilde{\varphi}_{hrs}(1-1/\sigma_h)} \qquad \perp \tilde{q}_{hrs} \tag{5}$$

$$c_{hr}f_{hrs} = \tilde{p}_{hrs}\tilde{q}_{hrs}\frac{(a+1-\sigma_h)}{a\sigma_h} \qquad \perp N_{hrs} \tag{6}$$

$$c_{hr}\delta f_{hr}^s = \sum_s \tilde{p}_{hrs}\tilde{q}_{hrs}\frac{(\sigma_h-1)}{a\sigma_h}\frac{N_{hrs}}{M_{hr}} \qquad \perp M_{hr} \tag{7}$$

$$\tilde{\varphi}_{hrs} = b\left(\frac{a}{a+1-\sigma_h}\right)^{1/(\sigma_h-1)}\left(\frac{N_{hrs}}{M_{hr}}\right)^{-1/a} \qquad \perp \tilde{\varphi}_{hrs} \tag{8}$$

$$Y_{hr} = \delta f_{hr}^s M_{hr} + \sum_s N_{hrs}\left(f_{hrs} + \frac{\tau_{hrs}q_{hrs}}{\tilde{\varphi}_{hrs}}\right) \qquad \perp c_{hr} \tag{9}$$

Equations (1), (2), and (3) – (9) form a square system of dimension $[(5 \times R \times H) + (4 \times R \times R \times H)]$ that establishes the Melitz trade equilibrium.

Exercises:

1. Convert model A3 into a one-factor one-sector general equilibrium [the Melitz (2003) model].

2. Use model B1 to show welfare equivalence across the Armington, Krugman, and Melitz models [Arkolakis et al. (forthcoming) equivalence].

3. Use model B1 to show the fragility of the Arkolakis et al. (forthcoming) equivalence. See the quote below for a hint.

4. Use model B1 to show different welfare results from tariffs in the Krugman and Melitz structures. What happens to this difference as the top-level intersectoral elasticity of substitution approaches one (Cobb-Douglas)? What happens to this difference as the variance on distribution of firm sizes gets large (as `esubm(h)` approaches `a(h)+1`)?

> These equivalence results are obtained in simplified structures where there is no change in entry (see Arkolakis et al., 2010). Our framework differs from Arkolakis et al. (2008) in at least three respects: a) we consider a multisector model, with resources moving between Melitz-style manufacturing and sectors with constant returns to scale, b) our model has multiple factors and intermediate inputs, and c) we examine experiments that remove observed revenue-generating tariffs rather than iceberg trade cost reductions. [Balistreri et al. (2011) *Journal of International Economics* 83(2) p.96].